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Throughput Modeling and Analysis of Random Access in Narrow-band Internet of Things

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***Abstract*—Narrow-band Internet of Things (NB-IoT) is one of the most promising technologies for low-power, wide-area, and low-traffic applications. In NB-IoT, random access is implemented in Media Access Control (MAC) layer to resolve the channel contention conflict among multiple User Equipments (UEs), and is crucial to the throughput performance of NB-IoT. Previous results in Long-Term Evolution (LTE) can not be directly applied due to specification differences. In this paper, we take the first attempt to systematically analyze the performance of random access in NB-IoT. First, after extensively studying the backoff mechanism, we characterize the probability that a UE initiates random access, the probability that a packet is transmitted successfully and the probability that a channel is busy. Then, we define each UE's buffer as a First-In-First-Out (FIFO) queue. We employ Markov chain to model retransmission number caused by collisions and the length of the queue simultaneously. By exploiting the characteristic of the steady-state distribution of the Markov chain, the above three probabilities in steady state can be obtained explicitly. Based on these probabilities, we calculate the system throughput in terms of UE number, packet generation rate, retransmission number and the length of the queue. Finally, we investigate the system throughput and conduct extensive simulations under various parameters, which validate our analysis.**

***Keywords*—Narrow-band Internet of Things, random access, Markov chain, First-In-First-Out queue**

I. Introduction

RECENT years have witnessed a dramatic increasing of intelligent devices [1]. It is reported by Ericsson [2] that there will be about 28 billion intelligent devices by 2021. This results in great demand in tailored communication technology [3]–[9]. Among a variety of communication technologies for Internet of Things (IoT), Narrow-band Internet of Things (NB-IoT), a promising technology for Low-Power Wide-Area applications proposed by the 3rd Generation Partnership Project (3GPP) in Release 13 [10], has been widely recognized as one of the most promising solutions. NB-IoT inherits from Long-Term Evolution (LTE), and has many distinctive advantages, including: 1) massive connections, 2) wide-area coverage, 3) low power consumption and 4) low cost [11]–[13].

Random access procedure is implemented in the Media Access Control (MAC) layer in NB-IoT to resolve the channel contention conflict among multiple User Equipments (UEs) [14]–[16]. Specifically, when there is data transmission demand, a UE selects the nearest eNodeB (eNB), and transmits the resources information in time and frequency domain to the eNB. The MAC layer operates the Radio Resource Control (RRC) [17] connection request in order to assist an eNB to efficiently allocate radio resources for the RRC connection setup complete message [18], [19]. If there are multiple channel requests, the eNB selects a UE randomly and allocates channel resources to the UE according to the UE's resources information. Then, the UE establishes the connection and transmits data to the eNB. The RRC, the data transmission, and the time synchronization [20] between UEs and eNB are feasible only when the random access procedure has been completed. Clearly, collisions can be extremely severe when there are a large number of users, leading to the underutilization of spatiotemporal spectrum resources. Therefore, it is of great importance to model and analyze the performance of random access as a function of participating UEs, which will be useful for system design and optimization of NB-IoT.

However, there are few works considering throughput modeling and analysis of random access in NB-IoT. Furthermore, previous results in LTE systems can not be directly applied to NB-IoT due to the following reasons. There are three coverage levels defined by Minimum Coupling Loss (MCL) [21] in NB-IoT, which covers a wider range than that in LTE, requiring more retransmission times during random access procedure. Due to the low-traffic rate and low-power consumption requirements, UEs in NB-IoT, like smart meters, have a finite buffer to cache data packets, while LTE requires higher data rate and UEs' buffers are larger. Thus, the length of buffers and more retransmission times need to be considered simultaneously in NB-IoT.

In this paper, we model and analyze the system throughput of random access in NB-IoT. Specifically, after extensively studying the backoff mechanism, we first characterize three probabilities, including the probability that a UE initiates random access, the probability that a packet is transmitted successfully and the probability that a channel is busy. Then, we define the finite buffer that each UE has as a First-In-First-Out (FIFO) queue. We employ Markov chain to model retransmission number and the length of the queue simultaneously. Different states in the Markov chain are defined by the retransmission caused by collisions and the length of the queue. By exploiting the characteristic of the steady-state distribution of the Markov chain, the above three probabilities in steady state can be obtained explicitly. Based on these probabilities in steady state, we then calculate the system throughput in

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terms of UE number, packet generation rate, retransmission number and the length of the queue. Finally, we investigate the system throughput and conduct extensive simulations under various parameters, which validate our analysis. Based on the performance of random access in NB-IoT, we can optimize random access procedure and thereby improve the throughput in the future. There are some models considering security in LTE, since NB-IoT inherits from LTE, based on such security models in LTE, we can reasonably focus on the throughput performance of random access in NB-IoT.

The contributions of this paper are three-fold:

1) To the best of our knowledge, this is an advanced work that takes the first attempt to consider the number of retransmission and the length of buffer in throughput modeling and analysis of random access in NB-IoT.
2) We characterize the probabilities that we need for calculating the system throughput under backoff mechanism. We employ Markov chain to model the queue and calculate the probabilities in steady state. Then we investigate the system throughput under various parameters.
3) To verify our model, we conduct extensive simulations under various parameters. The results verify our analysis.

The rest of the paper is organized as follows. In Section II, we discuss related works on NB-IoT and random access. Our system model is provided in Section III. In Section IV, we introduce the throughput modeling and analysis. We conduct extensive simulations to illustrate the impacts of different parameters and verify the analysis in Section V. We conclude this paper and show the future work in Section VI.

II. RELATED WORK

NB-IoT has recently attracted great attention, and performance analysis, positioning, system model design, etc. are hot topics in NB-IoT. For performance analysis, Lauridsen *et al.* [22] compared the coverage and capacity between LTE-Machine-to-Machine (LTE-M) and NB-IoT, showing that LTE-M has a requirement of 99.9% outdoor coverage and NB-IoT can provide more than 95% indoor coverage through simulations. Lauridsen *et al.* [23] considered Telenors commercial 2G, 3G, and 4G deployment and analyzed the coverage of GPRS, NB-IoT, LoRa, and SigFox in a 7,800 km^2 area, showing that NB-IoT provided the best coverage. For device positioning, Hu *et al.* [24] localized the devices deployed in an NB-IoT system using observed time-difference-of-arrival (OTDOA). For system model design, Ratasuk *et al.* [25] designed an NB-IoT system and analyzed it in terms of coverage, capacity, latency, and battery life. Mangalvedhe *et al.* [26] investigated the problems in the deployment of NB-IoT, and analyzed the high path-loss and the high interference in NB-IoT. Chen *et al.* [27] designed a tailored system including a development board and an NB-IoT platform for research. Petrov *et al.* introduced some relay nodes to help the UEs send data to eNBs and observed the probability of message loss by changing the number of relay nodes [28]. Miao *et al.* built a simulation model of NB-IoT in OPNET and validated the low-rate data transmission of NB-IoT [29].

Random access has been a hot research topic in LTE. Seo *et al.* [30] adopted Markov chain to analyze the initial random access and data transmission, characterized the number of terminals in steady state using the Equilibrium Point Analysis (EPA). Yang *et al.* [31] also used Markov chain to model the different states under the backoff mechanism in random access and obtained optimal parameters through simulations. Amirijoo *et al.* [32] modeled the self-optimization of the random access channel and evaluated the performance of the various parameters in the procedure. Ali *et al.* [33] proposed a random access model and resolved the preamble collisions. Mišić *et al.* [34] modeled the power ramping mechanism during random access in LTE/LTE-A in terms of preamble Signal to Interference plus Noise Ratio (SINR), preamble collisions, capture effect, which provided reasonable performance. Lin *et al.* [35] described the single-tone signal with frequency hopping designed for NB-IoT physical random access channel (NPRACH), and proposed corresponding receiver algorithms and time-of-arrival estimation. We summarize the related work in NB-IoT and random access in TABLE I.

The above mentioned works have not yet systematically investigated throughput modeling and analysis of random access in NB-IoT. To the best of our knowledge, we take the first attempt to consider a queue model for UEs' buffer in NB-IoT. Our model combines backoff mechanism and FIFO queue based on Markov chain, through which system throughput can be modeled and analyzed.

TABLE I: Related Work

Area	Work
NB-IoT	Coverage comparison [22], [23] System design and performance analysis [25] Positioning [24] Deployment [26] Development board and platform design [27] Relay nodes [28] Simulation model [29]
Random access	Performance analysis in LTE [30]–[32] Preamble collisions in LTE [33] Power ramping mechanism in LTE [34] Receiver algorithms and estimation in NB-IoT [35]

III. SYSTEM MODEL

We assume there are N UEs and one eNB channel in an NB-IoT system. Every UE has a buffer which stores K packets. The packet size in UEs' buffer is L and the transmission rate of data is v. λ packets are generated per second. When a UE needs to transmit a packet, it will initiate the following random access procedure.

1) **Random access preamble**. Random access preamble is the first step when UEs initiate random access procedure. In this step, the physical random access channel (PRACH) is configured according to the coverage levels, including normal coverage, extended coverage and extreme coverage. The corresponding MCL is 144/154/164dB. Every preamble sequence has a cyclic prefix (CP) and five symbols *1*. If the eNB does not receive the preamble sequences, UEs need to retransmit.
2) **Random access response (RAR)**. UEs start a timer called Random Access Response Window after transmitting the preamble sequences to the eNB. If the

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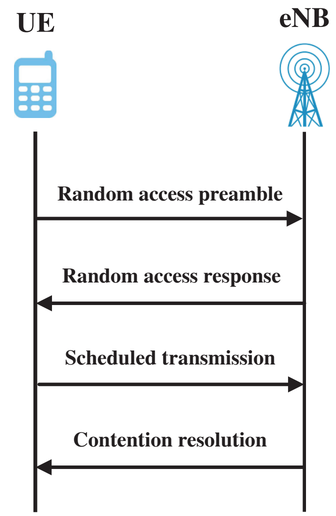

Fig. 1: Procedure of random access in NB-IoT.

UEs do not receive a response during this time, the sequences need to be retransmitted. After the eNB receives the preamble, it sends the RAR to the UEs, which includes the Random Access Radio Network Temporary Identity (RA-RNTI), the Timing Advance (TA), a temporary Cell Radio Network Temporary Identifier (C-RNTI) and some Msg3 scheduling messages. When more than one UE selects the same preamble sequences in step one, a collision happens. The conflicting UEs will receive the same RAR from eNB.

3) **Scheduled transmission**. The scheduled transmission, also called Msg3, contains the RRC connection, reconfiguration requests and Data Volume and Power Headroom Report (DPR). DPR consists of Power Headroom Report (PHR) and Buffer Status Report (BSR). UEs can be allocated with the corresponding channel resources according to DPR. Unless the UEs receive any response from eNB, they will redo this step.
4) **Contention resolution**. The contention resolution is also called Msg4. The eNB randomly selects a UE that completes step 3 to transmit Msg4. After receiving Msg4, the UE begins data transmission.

For ease of exposition, we make the following assumptions for the random access in NB-IoT:

1) In practice, when a collision happens, the UEs transmitting the same sequences in step 1 will receive the same RAR. The contention will be solved completely after step 4. Here we assume that retransmission is necessary when there is a collision in step 1. We do not consider any power or physical loss in random access procedure.
2) There is a cycle time T defined as the service time from the start of the current period to the start of the next period, including the maximum backoff time and the transmission time of a packet. The UEs that select the same delay parameter in backoff mechanism conflict with each other and need to wait for a new cycle. Once a UE occupies the channel, the other UEs have to wait for a new cycle, select a new delay parameter in backoff mechanism and initiate a new random access procedure.
3) If more than one UE selects the same delay parameter, collision happens. Then, the UEs need to select a new delay parameter randomly from 0 to the contention window size for retransmission. The contention window is the maximum delay in the backoff mechanism.
4) The length of a queue in NB-IoT is K, i.e., the queue can hold up to K packets. Different packet number represents different queue state. When the queue is empty, the UE does not initiate a random access procedure, i.e., the UE does not conflict with other UEs. The packets' arrivals is a Poisson process with a rate of λ.
5) The probability that a UE successfully transmits a data packet is equal to the probability that the UE occupies an available channel successfully. Once a UE successfully transmits a packet, it releases the channel. If the queue is empty, the UE becomes idle. When there is a new packet needs to be transmitted, the UE initiates a new random access procedure.

Some important notations used in our model are listed in TABLE II. Based on the assumptions above, we consider a fundamental investigation of throughput modeling and analysis of random access in NB-IoT under different parameters.

TABLE II: Notations

Symbol	Definition
N	Number of UEs
R	Maximum retransmission number
W	Number of delay parameters in backoff mechanism
$slot$	Slot time in backoff mechanism
λ	Packet generation rate (packets per second)
K	Length of queue at NB-IoT UEs
L	Packet size
v	Transmission rate of data
C_n	The probability that n UEs are contending for a channel
P_e	The probability that a queue is empty
P_s	The probability that a packet is successfully transmitted
P_b	The probability that a channel is busy
a_k	The probability that k packets arrive at the buffer
T	Cycle time
T_{packet}	Transmission time of a packet
P	State transition matrix
τ	System throughput

IV. THROUGHPUT MODELING AND ANALYSIS

In this section, we propose the system throughput modeling and analysis. In order to achieve the expression of system throughput, we design three steps in our model, including probabilities characterization under backoff mechanism, queue model under Markov chain and the system throughput.

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A. Probabilities Characterization under Backoff Mechanism

We first quantify the probability that a FIFO queue is empty as P_e, i.e., the probability that a UE does not initiate random access procedure, and the probability that a packet is transmitted successfully, defined by P_s. When two or more UEs initiate random access procedure and select the same delay parameter, collision happens. The UEs then wait for the next cycle and select arbitrary delay parameters randomly again. The conflicting UEs' retransmission number increase by one. If collision happens again or it reaches the maximum of retransmission, the UEs will redo the delay process till there is no collision in the system. During the whole procedure, the number of packets in a UE buffer still changes dynamically. If the maximum retransmission number is reached, the UEs drop the head-of-queue packets. If the queue is full of packets, the new incoming packets will be dropped. After transmitting a packet to eNB successfully, the UE releases the channel. When new packets stored in the buffer come, the UE initiates a new random access procedure.

If one UE (the observed UE) wants to occupy a channel to transmit a packet, the probability that n UEs out of the other $(N-1)$ UEs also have a non-empty queue is

$$\begin{aligned} C_n &= \binom{N-1}{n} \cdot P_e^{N-1-n} \cdot (1-P_e)^n \\ &= \frac{(N-1)!}{n!(N-1-n)!} \cdot P_e^{N-1-n} \cdot (1-P_e)^n. \end{aligned} \tag{1}$$

The probability that the observed UE occupies a channel and transmits a packet successfully means that all n UEs select a larger delay parameter in backoff mechanism than that of the observed UE, which is defined as

$$s_n = \sum_{\omega=1}^{W} \frac{1}{W} \left(\frac{W-\omega}{W} \right)^n. \tag{2}$$

When there is a UE transmitting a packet, that means the channel is busy and the other UEs can not access the channel, which means that at least one UE in n selects a smaller delay parameter than that of the observed UE. The probability that at least one UE in n selects a smaller delay parameter is

$$b_n = \sum_{\omega=1}^{W} \frac{1}{W} \left(1 - \left(\frac{W-\omega}{W} \right)^n \right), \tag{3}$$

when the channel is busy, the other UEs contending for the channel need to wait for a new cycle to start a new random access procedure.

Then, we obtain the probability P_s that a packet is transmitted successfully and the probability P_b that a channel is busy, which are functions of P_e, shown as Equation (4) and Equation (5), respectively.

From the probabilities above, the relationship between P_s and P_e, and the relationship between P_b and P_e can be obtained. In order to calculate them and investigate the system throughput, we introduce the FIFO queue model.

$$\begin{aligned} P_s &= f(P_e) \\ &= \sum_{n=0}^{N-1} C_n \cdot s_n \\ &= \sum_{n=0}^{N-1} \left(\frac{(N-1)!}{n!(N-1-n)!} \cdot P_e^{N-1-n} \cdot (1-P_e)^n \right) \cdot \\ &\quad \left(\sum_{\omega=1}^{W} \frac{1}{W} \left(\frac{W-\omega}{W} \right)^n \right), \end{aligned} \tag{4}$$

$$\begin{aligned} P_b &= g(P_e) \\ &= \sum_{n=0}^{N-1} C_n \cdot b_n \\ &= \sum_{n=0}^{N-1} \left(\frac{(N-1)!}{n!(N-1-n)!} \cdot P_e^{N-1-n} \cdot (1-P_e)^n \right) \cdot \\ &\quad \left(\sum_{\omega=1}^{W} \frac{1}{W} \left(1 - \left(\frac{W-\omega}{W} \right)^n \right) \right). \end{aligned} \tag{5}$$

B. Queue Model under Markov Chain

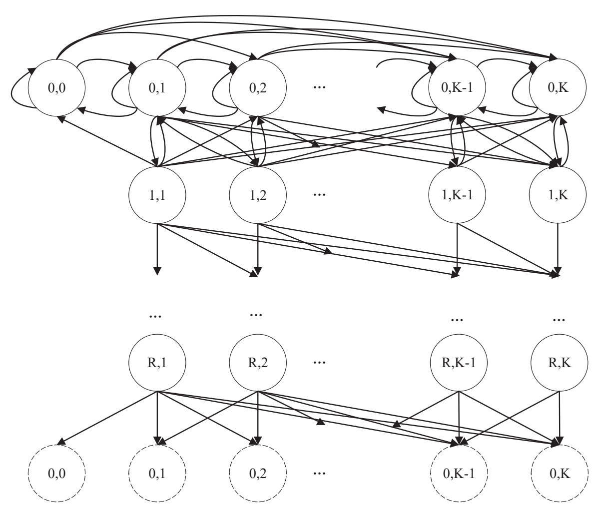

Fig. 2: The Markov chain for queue in NB-IoT.

The discrete-time Markov chain for FIFO queue of the observed UE is shown in Fig. 2. The length of the queue is K, thus the queue have $(K+1)$ states, from 0 packets to K packets. The queue length might be changed T by T. During the retransmission, there are still new packets coming. Thereby, both the maximum retransmission number R and the queue length K need to be considered. We define (r,k) as the different states in this Markov chain, $r \in [0,R]$, $k \in [0,K]$. The Markov chain has $(K \cdot (R+1)+1)$ states. The probability that k packets are generated in the duration T is a_k,

$$a_k = \frac{e^{-\lambda T}(\lambda T)^k}{k!}, \tag{6}$$

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where T is the cycle time that the system synchronizes all the UEs, which is defined as the interval from the start of the current period to the start of the next period. The time that a UE successfully occupies a channel and transmits a packet contains the delay time that the UE selects in backoff mechanism and the transmission time of a packet. We define T as the maximum delay time in backoff mechanism plus the transmission time of a packet. T is also the average service time between two different states in the Markov chain,

$$T = T_{packet} + W \cdot slot, \tag{7}$$

where T_{packet} is defined as the transmission time of a packet, W is defined as the number of delay parameters in backoff mechanism. The period of delay time in backoff mechanism is divided into slots, thus contention window (i.e., the maximum delay time in backoff mechanism) is $W \cdot slot$.

From Equations (6) and (7) above, a_k can be calculated. We denote $a_{\geq k}$ as the probability that there arrives no less than k packets at the NB-IoT buffer,

$$a_{\geq k} = \begin{cases} 1, & k \leq 0, \\ 1 - \sum_{i=0}^{k-1} a_i, & k > 0. \end{cases} \tag{8}$$

From Equations (6) to (8), the state transition matrix P of the Markov chain can be obtained as follows,

$$P_{(0,0)\to(0,0)} = a_0 \tag{9}$$

$$P_{(0,0)\to(0,m)} = a_m, \quad m \in [0, K-1] \tag{10}$$

$$P_{(0,0)\to(0,K)} = a_{\geq K} \tag{11}$$

$$P_{(0,m)\to(0,m-1)} = a_0 \cdot P_s, \quad m \in [1, K] \tag{12}$$

$$P_{(0,m)\to(0,n)} = a_{n-m+1} \cdot P_s, \quad m \in [1, K-1], n \in [m, K-1] \tag{13}$$

$$P_{(0,m)\to(0,K)} = a_{\geq K-m+1} \cdot P_s, \quad m \in [1, K] \tag{14}$$

$$P_{(r,m)\to(r,n)} = 0, \quad r \in [1, R], m \in [1, K-1], n \in [1, K-1] \tag{15}$$

$$P_{(r,m)\to(r,K)} = 0, \quad r \in [1, R], m \in [1, K] \tag{16}$$

$$P_{(r,m)\to(r+1,n)} = a_{n-m} \cdot (1 - P_s), \quad r \in [0, R-1], m \in [1, K-1], n \in [m, K-1] \tag{17}$$

$$P_{(r,m)\to(r+1,K)} = a_{\geq K-m} \cdot (1 - P_s), \quad r \in [0, R-1], m \in [1, K] \tag{18}$$

$$P_{(r,m)\to(0,n)} = a_{n-m+1} \cdot P_s, \quad r \in [1, R-1], m \in [1, K], n \in [m-1, K-1] \tag{19}$$

$$P_{(r,m)\to(0,K)} = a_{\geq K-m+1} \cdot P_s, \quad r \in [1, R-1], m \in [1, K] \tag{20}$$

$$P_{(R,m)\to(0,n)} = a_{n-m+1}, \quad m \in [1, K], n \in [m-1, K-1] \tag{21}$$

$$P_{(R,m)\to(0,K)} = a_{\geq K-m+1}, \quad m \in [1, K]. \tag{22}$$

From the transition matrix, in Equations (9) – (11), the probabilities only depend on the new arrival packets, since the packets' arrivals turn the idle UE to be active. Equations (12) – (14) show the different states in the first transmission after initiating the random access procedure. If a packet is successfully transmitted in the first transmission, the length of the queue reduces by one. Or in the meanwhile, a number of new packets come, the length of the queue increases. If the r-th retransmission is successful, the state transfers from (r, k) to $(0, k)$. On the contrary, when the r-th retransmission fails, the state transfers from (r, k) to $(r + 1, k)$. Thus the probabilities in Equations (15) and (16) are 0. In Equations (17) and (18), collision happens and UEs need to retransmit the packet and the packet is still in its buffer, thus the conflicting UEs' retransmission number increase by one. Equations (19) and (20) show the r-th retransmission is successful. In Equations (21) and (22), no matter whether the R-th retransmission is successful or not, the state transfers from (R, k) to $(0, k)$ with new coming packets arriving, or the UE will be idle if the queue becomes empty. Since the Markov chain is irreducible and aperiodic, a unique steady state can be obtained. We assume that $\pi = [\pi(0,0), \pi(0,1), \cdots, \pi(0,K); \pi(1,1), \cdots, \pi(1,K); \cdots; \pi(R,1), \cdots, \pi(R,K)]$ is the steady state of the Markov chain. From the equation $\pi = \pi P$, we can obtain $\pi(0,0)$, i.e., the probability P_e that the queue is empty in steady state.

P_e is a function of P_s from the queue model, and from the probabilities characterized under backoff mechanism, P_s is a function of P_e. Combing (4) with the steady-state distribution of the Markov chain, P_e and P_s in steady state can be calculated. Then P_b can be calculated in numerical results from Equation (5).

C. The System Throughput

In our model, the system throughput τ, is the number of packets that UEs in the system successfully transmit per second. Using the number of UEs and the probability P_e that a queue is empty, we can obtain τ,

$$\tau = \frac{\tau'}{T}, \tag{23}$$

where

$$\tau' = \binom{N}{1}(1-P_e)P_e^{N-1} + \sum_{n=2}^{N}\left(\binom{N}{n}(1-P_e)^n P_e^{N-n}\right) \cdot \left(\sum_{\omega=1}^{W}\binom{n}{1}\frac{1}{W}\left(\frac{W-\omega}{W}\right)^{n-1}\right), \tag{24}$$

T is the cycle time shown in (7), also the average service time between two different states in the Markov chain. The system throughput consists of two parts. One part is that only one UE initiates random access procedure and the UE successfully

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occupies the channel; the other part is that more than one UE initiates random access procedure and one of the UEs successfully occupies the channel.

V. Numerical Results and Validation

A. Simulation Settings

Here we verify our model via extensive simulations under various parameters. Firstly, according to the smart meter application, we define the packets size L as 512 $bytes$ and transmission rate of data v as 160 $kbps$. According to 3GPP Release 13 specification, the contention window is a value that the eNB selects from the set $\mathcal{T}$ = $\{0, 20, 30, 40, 60, 80, 120, 160, 240, 320, 480, 960\}$ ms and sends to UEs for delay in backoff mechanism. Combining the packets size with the transmission rate in NB-IoT, the time that a packet is transmitted is 25 ms. If we select a too large contention window in the scenario, the UEs may still delay for a long time after a packet is transmitted successfully. Thus, for practical consideration, we select 20 ms as the contention window to ensure the appropriate length of cycle time. In order to guarantee the compatibility with LTE, the slot of NB-IoT in time domain is composed of 7 Orthogonal Frequency Division Multiplexing (OFDM) symbols, which is 0.5 ms, thus $slot$ can be set as 0.5 ms. Combing the contention window with the $slot$, we then set W as 40. The maximum retransmission number is a number selected from the set $\mathcal{R}$ = $\{1, 2, 4, 8, 16, 32, 64, 128\}$ according to Release 13 specification, which is a varied parameter in this part. Finally, we set $N = 5$, $R = 8$, $W = 40$, $\lambda = 0.1$, $K = 10$ as the primary parameters. In order to calculate the system throughput in different scenarios, we set some of the parameters to constants and vary the remaining. Considering the following scenarios, some simulation and analysis results of the system throughput can be obtained.

1) Varying the UE number with different packet generation rate.
2) Varying the maximum retransmission number with different packet generation rate.
3) Varying the length of the queue with different packet generation rate.

Figures 3 to 5 show the analysis and simulation results of the system throughput in different scenarios, as detailed below.

B. Varying the UE Number with Different Packet Generation Rate

We set $W = 40$, $R = 8$, $K = 10$ and vary N with different λ. As Fig. 3 shows, the number of UEs has obvious effect on system throughput. When the number of UEs increases, the system throughput also increases. When λ is small, the system throughput increases slowly. When λ is larger, the system throughput increases rapidly at the beginning, reaches a peak and then gradually decreases. From Fig. 3, the maximum system throughput is a little more than 21 packets per second. When the number of UEs in the system is small, collisions seldom occur. When there are a large number of UEs, collisions are more likely to happen. We can assume that when the number of UEs tends to infinity, the throughput drops to 0. However, the system can still maintain a certain throughput when there are a large number of users, which is consistent with the large connections in NB-IoT. Therefore, it is significant to find an appropriate N based on different packet generation rate. When the packet generation rate in an NB-IoT system is small, a large number of UEs can be supported. If the packet generation rate is large, the system can support a few UEs. Simultaneously, knowing the number of UEs in an NB-IoT system, we can obtain the packet generation rate that the system supports. When there are over 50 UEs contending for one channel, the system supports $\lambda = 1$, which can still maintain a high throughput, as shown in Fig. 3.

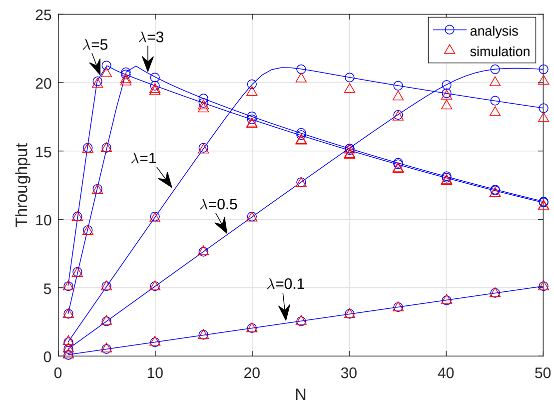


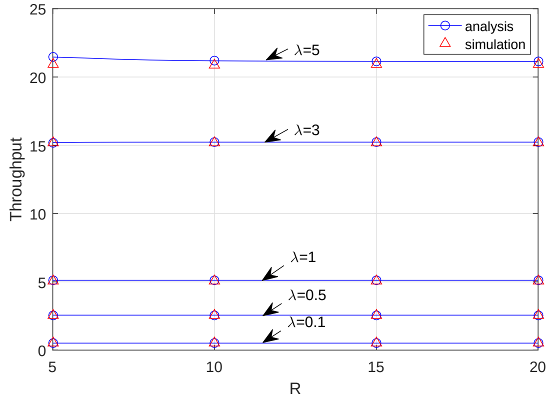
Fig. 3: Comparison of the system throughput varying N with different λ between analysis and simulation.

C. Varying the Maximum Retransmission Number with Different Packet Generation Rate

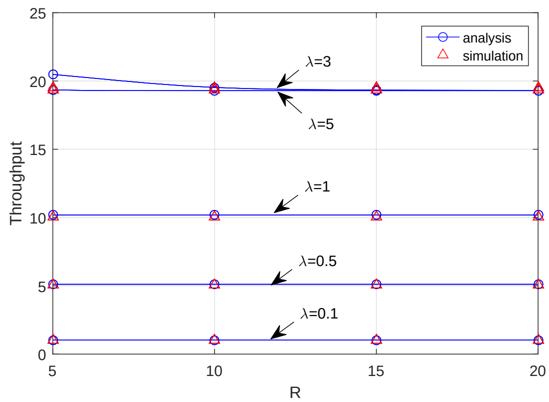
We set $N = 5, 10, 20$, $W = 40$, $K = 10$ and vary R with different λ. As Fig. 4 shows, the retransmission number has slight influence on the system throughput. When the UE number and the packet generation rate are small, the system throughput can be stable with R increasing. However, when N or λ is larger, the system throughput experiences a fluctuation from the beginning and then remains stable gradually. Due to the randomness in backoff mechanism, the analysis may be unstable when R is small, which may fluctuate in the beginning with a large λ. However, we run the simulation and wait for its stabilization before calculating the throughput, through which the throughput is relatively stable, thus the fluctuation in simulation is not obvious. In NB-IoT systems, retransmission is a means of supporting wide coverage, thus R is not suitable to be selected too small.

TABLE III: The probability P_b that a channel is busy

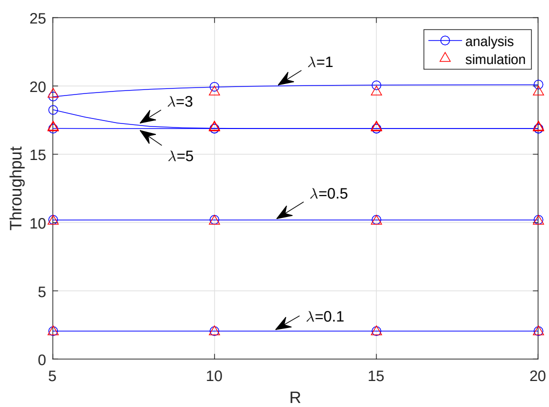
P_b	λ = 0.1	λ = 0.5	λ = 1	λ = 3	λ = 5
N=5	0.0089	0.0456	0.0937	0.3286	0.7620
N=10	0.0202	0.1050	0.2230	0.8551	0.8869
N=20	0.0430	0.2344	0.5878	0.9342	0.9365
N=30	0.0662	0.3879	0.8562	0.9520	0.9527
N=40	0.0898	0.5980	0.9048	0.9602	0.9605
N=50	0.1140	0.7917	0.9262	0.9648	0.9650



(a) N=5

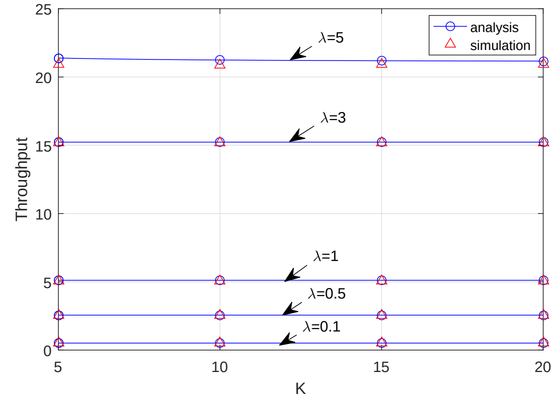


(b) N=10

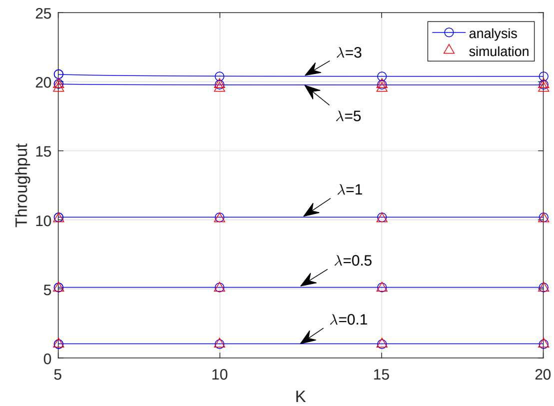


(c) N=20

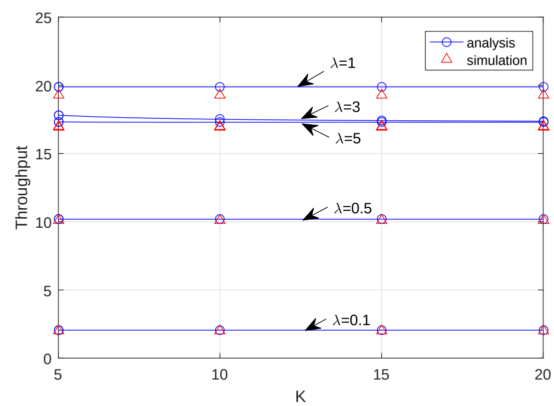
Fig. 4: Comparison of the system throughput varying R with different λ between analysis and simulation.



(a) N=5



(b) N=10



(c) N=20

Fig. 5: Comparison of the system throughput varying K with different λ between analysis and simulation.

D. Varying the Length of the Queue with Different Packet Generation Rate

We set $N = 5, 10, 20$, $R = 8$, $W = 40$ and vary K with different λ. As Fig. 5 shows, the length of the queue influences the system throughput slightly. When K increases, the system throughput basically remains stable with different N or different λ. When λ increases, the system throughput also increases. Simulation and analysis are largely consistent here.

E. The Probability that the Channel is Busy

Based on the probability P_e that a queue is empty, the probability P_b that a channel is busy can be calculated. Different λ and N have different degrees of impacts on P_b, which is shown in TABLE III. When λ or N is larger, the channel is more likely to be busy.

VI. Conclusion and Future Work

In this paper, based on backoff mechanism in NB-IoT systems, we characterize the probabilities that a UE initiates random access, a packet is transmitted successfully and a channel is busy. Then, we define the UEs' buffer as a FIFO queue. To obtain the three probabilities in steady state, we employ Markov chain to model the length of the queue and retransmission. Based on the obtained probabilities, we investigate the system throughput in terms of UE number, packet generation rate, retransmission number and the length of the queue. Finally, We conduct extensive simulations to validate our model, which show consistent results with analysis. With various parameters, we obtain the appropriate parameters in the system and achieve the performance of random access in NB-IoT. We can optimize the random access procedure and improve the system throughput based on the proposed model. In NB-IoT systems, system throughput, the latency that a packet is transmitted successfully, and the probability of packet loss are key metrics in Quality of Service (QoS), which are important for system performance. In order to guarantee QoS, we should guarantee the system throughput, and reduce latency and packet loss. In the future, we may analyze the latency that a packet is transmitted successfully and the probability of packet loss, and we can analyze the system performance more comprehensively through these parameters.

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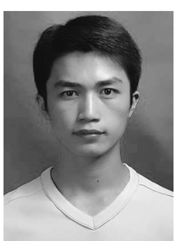

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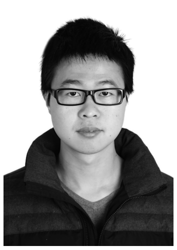

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